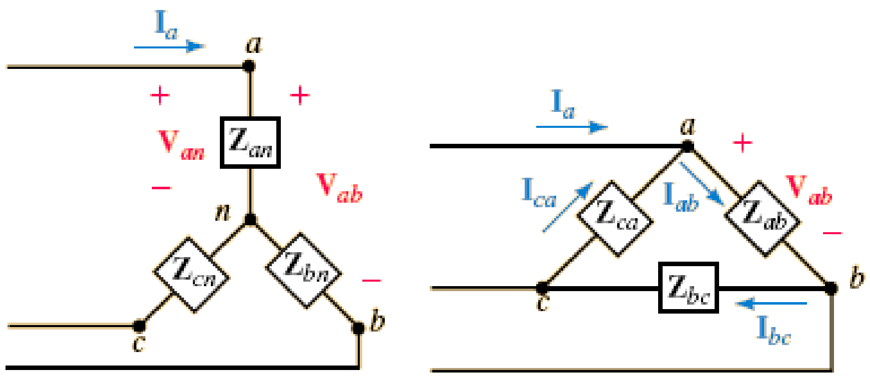
Algunos ejemplos resueltos

Para un sistema balanceado (resumen)



(a) Y connection (b) Δ connection

$$\mathbf{V}_{ab} = \sqrt{3}\mathbf{V}_{an}\angle 30^\circ \qquad \mathbf{I}_a = \sqrt{3}\mathbf{I}_{ab}\angle -30^\circ$$

$$\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_{an} \qquad \mathbf{I}_{ab} = \mathbf{V}_{ab}/\mathbf{Z}_{ab}$$

$$\mathbf{Z}_{an} = \mathbf{Z}_{bn} = \mathbf{Z}_{cn} \qquad \mathbf{Z}_{ab} = \mathbf{Z}_{bc} = \mathbf{Z}_{ca}$$

$$\text{Generator } \mathbf{E}_{AB} = \sqrt{3}\mathbf{E}_{AN}\angle 30^\circ$$

Active Power to a Balanced Wye Load

First, consider a Y load (Figure 23–22). The power to any phase as indicated in (b) is the product of the magnitude of the phase voltage V_ϕ times the magnitude of the phase current I_ϕ times the cosine of the angle θ_ϕ between them. Since the angle between voltage and current is always the angle of the load impedance, the power per phase is

$$P_\phi = V_\phi I_\phi \cos\theta_\phi \quad \text{(W)} \tag{23–10}$$

where θ_ϕ is the angle of $\mathbf{Z}_\phi$. Total power is

$$P_T = 3P_\phi = 3V_\phi I_\phi \cos\theta_\phi \quad \text{(W)} \tag{23–11}$$

It is also handy to have a formula for power in terms of line quantities. For a Y load, $I_\phi = I_L$ and $V_\phi = V_L/\sqrt{3}$, where I_L is the magnitude of the line current and V_L is the magnitude of the line-to-line voltage. Substituting these relations into Equation 23–11 and noting that $3/\sqrt{3} = \sqrt{3}$ yields

$$P_T = \sqrt{3}V_L I_L \cos\theta_\phi \quad \text{(W)} \tag{23–12}$$

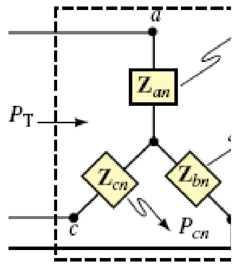
This is a very important formula and one that is widely used. Note carefully, however, that θ_ϕ is the angle of the load impedance and not the angle between V_L and I_L.

Power per phase can also be expressed as

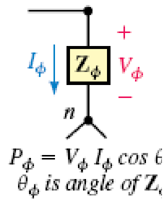
$$P_\phi = I_\phi^2 R_\phi = V_R^2/R_\phi \quad \text{(W)} \tag{23–13}$$

where R_ϕ is the resistive component of the phase impedance and V_R is the voltage across it. Total power is thus

$$P_T = 3I_\phi^2 R_\phi = 3V_R^2/R_\phi \quad \text{(W)} \tag{23–14}$$



(a) $P_T = P_{an} + P_{bn} + P$



(b)

FIGURE 23–22 For a ba
$P_\phi = P_{an} = P_{bn} = P_{cn}$.

Reactive Power to a Balanced Wye Load

Equivalent expressions for reactive power are

$$Q_\phi = V_\phi I_\phi \sin\theta_\phi \quad \text{(VAR)} \tag{23–15}$$

$$= I_\phi^2 X_\phi = V_X^2/X_\phi \quad \text{(VAR)} \tag{23–16}$$

$$Q_T = \sqrt{3} V_L I_L \sin\theta_\phi \quad \text{(VAR)} \tag{23–17}$$

where X_ϕ is the reactive component of $\mathbf{Z}_\phi$ and V_X is the voltage across it.

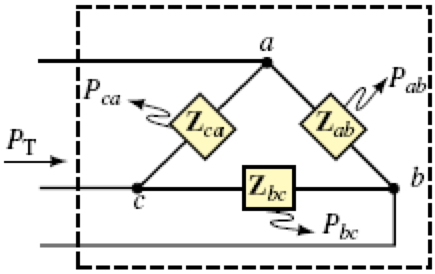
Apparent Power

$$S_\phi = V_\phi I_\phi = I_\phi^2 Z_\phi = \frac{V_\phi^2}{Z_\phi} \quad \text{(VA)} \tag{23–18}$$

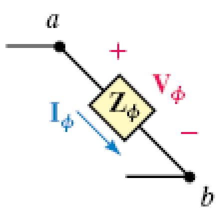
$$S_T = \sqrt{3} V_L I_L \quad \text{(VA)} \tag{23–19}$$

Power Factor

$$F_p = \cos\theta_\phi = P_T/S_T = P_\phi/S_\phi \tag{23–20}$$



(a) $P_T = P_{ab} + P_{bc} + P_{ca} = 3\,P_\phi$.



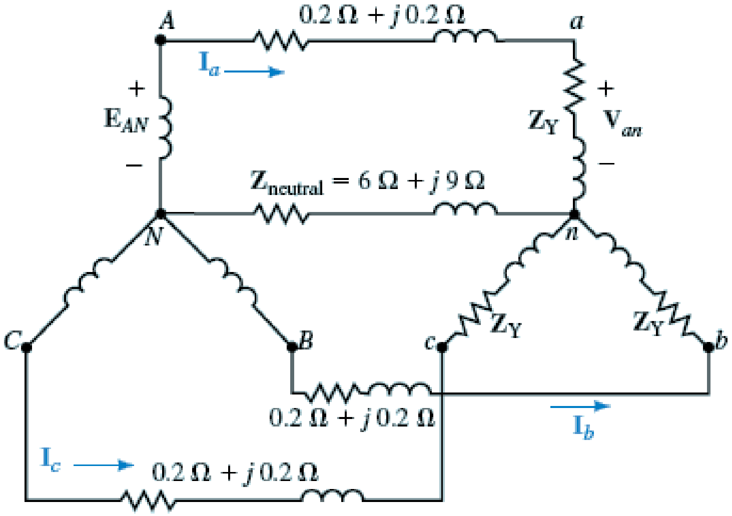
(b) $P_\phi = V_\phi I_\phi \cos\theta_\phi$
θ_ϕ is angle of load impedance

FIGURE 23–24 For a balanced Δ, $P_\phi = P_{ab} = P_{bc} = P_{ca}$.

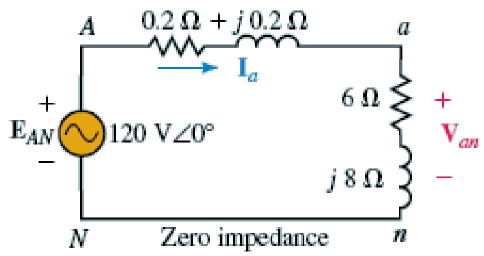
Active power	$P_\phi = V_\phi I_\phi \cos\theta_\phi = I_\phi^2 R_\phi = \dfrac{V_R^2}{R_\phi}$
	$P_T = \sqrt{3} V_L I_L \cos\theta_\phi$
Reactive power	$Q_\phi = V_\phi I_\phi \sin\theta_\phi = I_\phi^2 X_\phi = \dfrac{V_x^2}{X_\phi}$
	$Q_T = \sqrt{3} V_L I_L \sin\theta_\phi$
Apparent power	$S_\phi = V_\phi I_\phi = I_\phi^2 Z_\phi = \dfrac{V_\phi^2}{Z_\phi}$
	$S_T = \sqrt{3} V_L I_L$
Power factor	$F_p = \cos\theta_\phi = \dfrac{P_T}{S_T} = \dfrac{P_\phi}{S_\phi}$
Power triangle	$\mathbf{S}_T = P_T + jQ_T$

EXAMPLE 23–6 For Figure 23–16, $\mathbf{E}_{AN} = 120\text{ V}\angle 0°$.

a. Solve for the line currents.
b. Solve for the phase voltages at the load.
c. Solve for the line voltages at the load.



(a) $\mathbf{Z}_Y = 6\ \Omega + j\,8\ \Omega$



(b) Single phase equivalent. Since the neutral conductor in (a) carries no current, its impedance has no effect on the solution

Solution

a. Reduce the circuit to its single-phase equivalent as shown in (b).

$$\mathbf{I}_a = \frac{\mathbf{E}_{AN}}{\mathbf{Z}_T} = \frac{120\angle 0°}{(0.2 + j0.2) + (6 + j8)} = 11.7\text{ A}\angle -52.9°$$

Therefore,

$$\mathbf{I}_b = 11.7\text{ A}\angle -172.9° \quad \text{and} \quad \mathbf{I}_c = 11.7\text{ A}\angle 67.1°$$

b. $\mathbf{V}_{an} = \mathbf{I}_a \times \mathbf{Z}_{an} = (11.7\angle -52.9°)(6 + j8) = 117\text{ V}\angle 0.23°$

Thus,

$$\mathbf{V}_{bn} = 117\text{ V}\angle -119.77° \quad \text{and} \quad \mathbf{V}_{cn} = 117\text{ V}\angle 120.23°$$

c. $\mathbf{V}_{ab} = \sqrt{3}\mathbf{V}_{an}\angle 30° = \sqrt{3} \times 117\angle(0.23° + 30°) = 202.6\text{ V}\angle 30.23°$

Thus,

$$\mathbf{V}_{bc} = 202.6\text{ V}\angle -89.77° \quad \text{and} \quad \mathbf{V}_{ca} = 202.6\text{ V}\angle 150.23°$$

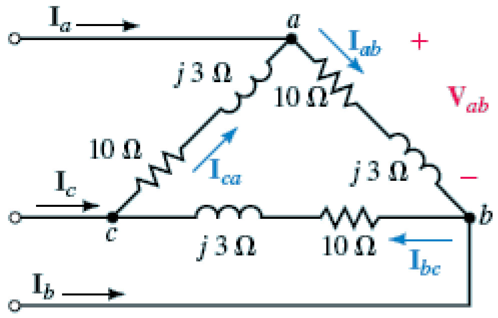
Note the phase shift and voltage drop across the line impedance. Note also that the impedance of the neutral conductor plays no part in the solution, since no current passes through it because the system is balanced.

EXAMPLE 23–5 Suppose $\mathbf{V}_{ab} = 240\text{ V}\angle 15°$ for the circuit of Figure 23–13.

a. Determine the phase currents.

b. Determine the line currents.

c. Sketch the phasor diagram.



EWB FIGURE 23–13

Solution

a. $\mathbf{I}_{ab} = \dfrac{\mathbf{V}_{ab}}{\mathbf{Z}_{ab}} = \dfrac{240\angle 15°}{10 + j3} = 23.0\text{ A}\angle -1.70°$

Thus,

$$\mathbf{I}_{bc} = 23.0\text{ A}\angle -121.7° \quad \text{and} \quad \mathbf{I}_{ca} = 23.0\text{ A}\angle 118.3°$$

b. $\mathbf{I}_a = \sqrt{3}\mathbf{I}_{ab}\angle -30° = 39.8\text{ A}\angle -31.7°$

Thus,

$$\mathbf{I}_b = 39.8\text{ A}\angle -151.7° \quad \text{and} \quad \mathbf{I}_c = 39.8\text{ A}\angle 88.3°$$

c. Phasors are shown in Figure 23–14.

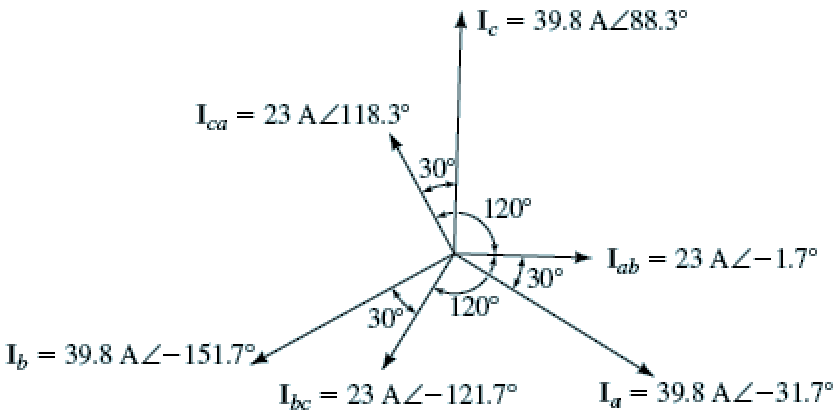


FIGURE 23–14

EXAMPLE 23–12 Determine per phase and total power (active, reactive, and apparent) for Figure 23–25. Use $V_\phi = 207.8$ V in order to compare results.

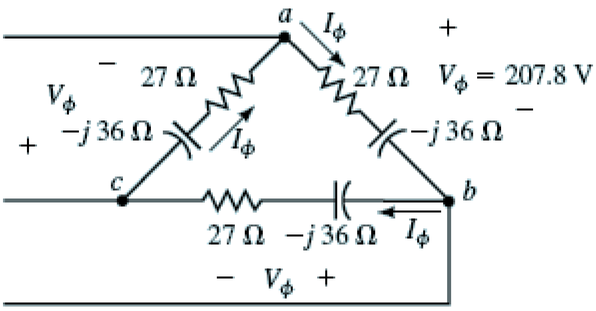


FIGURE 23–25

Solution

$$\mathbf{Z}_\phi = 27 - j36 = 45\ \Omega\angle{-53.13°}, \quad \text{so} \quad \theta_\phi = -53.13°$$

$$V_\phi = 207.8\ \text{V} \quad \text{and} \quad I_\phi = V_\phi/Z_\phi = 207.8\ \text{V}/45\ \Omega = 4.62\ \text{A}$$

$$P_\phi = V_\phi I_\phi \cos\theta_\phi = (207.8)(4.62)\cos(-53.13°) = 576\ \text{W}$$

$$Q_\phi = V_\phi I_\phi \sin\theta_\phi = (207.8)(4.62)\sin(-53.13°) = -768\ \text{VAR}$$

$$= 768\ \text{VAR (cap.)}$$

$$S_\phi = V_\phi I_\phi = (207.8)(4.62) = 960\ \text{VA}$$

$$P_\text{T} = 3P_\phi = 3(576) = 1728\ \text{W}$$

$$Q_\text{T} = 3Q_\phi = 3(768) = 2304\ \text{VAR (cap.)}$$

$$S_\text{T} = 3S_\phi = 3(960) = 2880\ \text{VA}$$

Note that the results here are the same as for Example 23–11. This is to be expected since the load of Figure 23–23 is the Y equivalent of the Δ load of Figure 23–25.